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ACOUSTICS AND STABILITY OF FLUID FLOW IN A PERIODIC ELASTIC STRUCTURE

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A fluid flows horizontally through a fluid–structure system comprising alternating elastic solid and liquid constituents arranged periodically in the vertical direction. An exact analysis is performed to consider the existence and stability of small acoustic waves and disturbances. The presence of the flow introduces the possibility of flow-induced flutter. Unstable waves are generally possible for M of order unity, M being the Mach number relative to the speed of shear waves in the solid. Instabilities can appear for much lower values for antisymmetric flexural type motion. In that case it is found that a critical wavenumber exists, indicating that the layered system is inherently unstable to long wavelength disturbances.

1. INTRODUCTION

Fluid-saturated permeable elastic solids exhibit distinct phenomena not seen in either the solid or the liquid phase. The most interesting acoustic feature is the appearance of the Biot "slow" wave, predicted by Biot (1956) and observed by Plona (1980) in water-saturated sintered beads. Similar acoustic effects have since been seen in other systems, including layered periodic solid–liquid configurations (Plona *et al.* 1987). The periodicity of the layered structure permits a more precise theoretical analysis than for a disordered system, and very good agreement has been observed between theory (Rytov 1956; Schoenberg 1983) and experiment (Plona *et al.* 1987). The acoustics of such layered solid–liquid structures is of great interest in various applications relating to physics, mechanical engineering, Earth science, etc. (Brekhovskikh 1981), because the structure displays the essential features of a realistic permeable elastic medium. In all previous studies of this system it was assumed that both the solid and fluid constituents are at rest in the "ground" equilibrium configuration. On the other hand, it is well known that sufficiently fast fluid motion through a deformable solid structure can destabilize the system, causing flutter-like phenomena (Bolotin 1963).

Flutter has been studied extensively using engineering theories of plates and shells for the solid phase, combined with hydrodynamic theories for the liquid, which are sometimes based on the hypothesis of plane cross-sections, rather than with the help of much more reliable theories of bulk hydrodynamics and elasticity (Fung 1955; Bolotin 1963). Indeed, there is a large amount of literature on the important problem of flow stabilization using compliant coatings and structures; see Riley *et al.* (1988) for a review. The main issue is how to suppress or delay the onset of instabilities associated with a plethora of possible wave types, some associated primarily with viscosity, others with the structure. Here we ignore effects of viscosity and assume the simplest type of inviscid flow. In this regard the present study is more closely related to those of Brazier-Smith & Scott (1984) and of Crighton & Owell (1991), who considered an

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isolated elastic plate in a uniform inviscid flow. Thin plate equations (Kirchhoff theory) were used to model the structure and the fluid was assumed to be incompressible. In this study we make no approximations other than the inviscid fluid and the uniform flow assumptions. The solid and fluid are modeled precisely otherwise.

The purpose of this paper is to report on new results and physical phenomena caused by the non-small relative velocity of the fluid propagating between solid elastic layers. New, flutter-like instabilities are found and their asymptotic properties identified. Some of these effects are a direct consequence of the flow, and vanish in the equilibrium, no-flow state. The study of acoustic disturbances and flutter-like phenomena in the presence of flow requires a nonlinear basis, and hence our study begins with an exact nonlinear formulation. We then linearize the equations and interface boundary conditions in the vicinity of a stationary state in which parallel isotropic elastic layers are undeformed, while the fluid moves with the constant velocity V^0. Our objective is to examine the dependence on V^0 of the dispersion equations of acoustic waves in the small vicinity of the steady, uniform flow configuration. No attempt is made here to explore what is undoubtedly a rich field for numerical study. Rather, we report exact results and various asymptotic limits, such as long- and short-wavelengths, thin solid regions, and joined half-spaces.

2. DISPERSION RELATIONS

2.1. Nonlinear Equations

Let us consider a periodically layered medium consisting of alternating solid and fluid layers. The layers are infinite in the lateral direction, and the periodicity extends indefinitely in the vertical direction. The fluid occupying the gaps between neighboring solid layers is assumed to be inviscid. We choose the Eulerian description of continua in order to simplify exactly the formulation of the interface boundary conditions. We emphasize that the relative displacements of the fluid and solid constituents are not small, generally speaking. We denote by x^i the Eulerian Cartesian coordinates, and assume that the undisturbed elastic layers lie parallel to the plane x^3 constant.

The bulk equations within the solid and fluid constituents and the interface boundary conditions are

$$\rho_s\left(\frac{\partial V_s^i}{\partial t} + V_s^j\nabla_j V_s^i\right) = \nabla_j P_s^{ji}, \tag{1a}$$

$$V_s^i = \frac{\partial U_s^i}{\partial t} + V_s^j\nabla_j U_s^i, \tag{1b}$$

$$\rho_f\left(\frac{\partial V_f^i}{\partial t} + V_f^j\nabla_j V_f^i\right) = -\nabla^i p_f, \tag{1c}$$

$$\frac{\partial \rho_f}{\partial t} + \nabla_i(\rho_f V_f^i) = 0, \tag{1d}$$

$$P_s^{ji}N_j = -p_f N^i, \tag{1e}$$

$$V_s^j N_j = V_f^j N_j, \tag{1f}$$

where ρ_s and ρ_f are the actual densities of the constituents, V_s^i and V_f^i are their velocities, U_s^i is the displacement of the solid, P_s^{ji} and p_f are the Cauchy stress tensor of the solid and the fluid pressure, respectively, and N^i is the unit normal of the interface. The system of equations (1) permits a stationary solution in which the solid layers are at rest and undeformed, whereas the fluid moves with constant velocity V^0 parallel to the elastic layers. Linearizing the system of equations (1) about this state yields the governing system for small disturbances, discussed next.

2.2. Linearization

We consider small dynamic motion superimposed on the state of uniform pressure and flow in the fluid, corresponding to fluid velocity V^0, fluid pressure p^0, and densities ρ_f^0 and ρ_s^0. The solid is also assumed to be in a state of uniform static initial stress, $P_s^{ji} = P_0^{ji}$, either hydrostatic ($P_0^{ji} = -p^0\delta^{ji}$) or otherwise, with the initial deformation homogeneous and defined by $U_s^i = U_0^i$. Thus, let

$$V_f^i = V^0\delta^{i1} + v_f^i, \qquad p_f = p^0 + p,$$
$$U_s^i = U_0^i + u_s^i, \qquad P_s^{ji} = P_0^{ji} + \sigma^{ji}, \tag{2}$$

where v_f^i and the remaining dynamic quantities (u_s^i, V_s^i, σ^{ji}, $\rho_s - \rho_s^0$, $\rho_f - \rho_f^0$, and p) are small. We consider motion in the $x^1 - x^3$ plane, where, as mentioned before, x^3 is the coordinate in the layering direction. Then equations (1) imply the following linearized equations:

$$\rho_s^0 \frac{\partial V_s^i}{\partial t} = \nabla_j \sigma^{ji}, \tag{3a}$$

$$V_s^i = \frac{\partial u_s^i}{\partial t}, \tag{3b}$$

$$\rho_f^0\left(\frac{\partial v_f^i}{\partial t} + V^0 \frac{\partial v_f^i}{\partial x^1}\right) = -\nabla^i p, \tag{3c}$$

$$\frac{\partial \rho_f}{\partial t} + \rho_f^0 \nabla_i v_f^i + V^0 \frac{\partial \rho_f}{\partial x^1} = 0, \tag{3d}$$

$$\sigma^{3i} = -p\delta^{3i}, \tag{3e}$$

$$V_s^3 + V^0 \frac{\partial u_s^3}{\partial x^1} = v_f^3. \tag{3f}$$

The equations for the solid phase, (3a,b), must be supplemented by a linear stress-strain relation for the small stress σ^{ji}, of the form

$$\sigma_{ji} = C_{jikl} \nabla^k u_s^l. \tag{4}$$

Therefore, equations (3a) and (3b) imply that U_s^i satisfies the usual equations of linear dynamic elasticity,

$$C_{jikl} \nabla^j \nabla^k u_s^l - \rho_s^0 \frac{\partial^2 u_s^i}{\partial t^2} = 0. \tag{5}$$

Equations (3c,d) for the fluid require an additional linear equation of state,

$$\rho_f - \rho_f^0 = \frac{p}{c_f^2}. \tag{6}$$

where c_f is the speed of sound. When combined with equations (3c) and (3d), this implies a convective wave equation for the small pressure,

$$\nabla_i \nabla^i p - \frac{1}{c_f^2}\left(\frac{\partial}{\partial t} + V^0 \frac{\partial}{\partial x^1}\right)^2 p = 0. \tag{7}$$

TABLE 1

Summary of wave speeds in the problem. The solid shear modulus is $\mu > 0$ and Poisson ratio $-1 < \nu < \frac{1}{2}$. For each speed, c_α, the dimensionless speed is $s_\alpha = c_\alpha / c_t$

Speed	Definition
System mode	$c = \omega / k$
Fluid acoustic	$c_f^2 = \mathrm{d}p/\mathrm{d}\rho_f\big\|_{\rho_f^0}$
Shear	$c_t^2 = \mu / \rho_s^0$
Longitudinal	$c_l^2 = c_t^2 2(1-\nu)/(1-2\nu)$
Plate	$c_p^2 = c_t^2 2/(1-\nu)$
Bending	$c_b^2 = c_t^2 2k^2 H_s^2/[3(1-\nu)]$

2.3. Traveling Wave Solutions

Let us consider "traveling-wave"-solutions of the linearized system, i.e. the solutions proportional to $\mathrm{e}^{\mathrm{i}(kx^1-\omega t)}$. There are two distinct types of solutions, which we call symmetric and antisymmetric, respectively. For the symmetric mode the shape of each layer remains symmetric with respect to its median. The dispersion equation for this mode is derived below as

$$\left(2-\frac{c^2}{c_t^2}\right)^2 \frac{\tanh(k\xi_t H_s)}{\tanh(k\xi_l H_s)} - 4\xi_l\xi_t + \frac{\rho_f^0}{\rho_s^0}\left(1-\frac{V^0}{c}\right)^2 \frac{\xi_l}{\xi_f}\frac{c^4}{c_t^4}\frac{\tanh(k\xi_t H_s)}{\tanh(k\xi_f H_f)} = 0. \tag{8}$$

Here we use the following notation: $c = \omega/k$, is the velocity of the traveling wave, $2H_s$ and $2H_f$ are the equilibrium thicknesses of the fluid and solid layers, respectively, ρ_s^0 and ρ_f^0 are the undisturbed densities, and

$$\xi_l = (1 - c^2/c_l^2)^{1/2}, \qquad \xi_t = (1 - c^2/c_t^2)^{1/2}, \qquad \xi_f = (1 - (c - V^0)^2/c_f^2)^{1/2}, \tag{9}$$

where c_t and c_l are the velocities of bulk transverse and longitudinal waves, respectively, within the undeformed solid layer, and c_f is the bulk sound velocity within the undisturbed fluid. The various wave speeds and associated dimensionless parameters are summarized in Table 1.

In the antisymmetric mode, opposite edge points of each layer have the same vertical velocity. The dispersion equation for the antisymmetric mode is the following:

$$\left(2-\frac{c^2}{c_t^2}\right)^2 \frac{\tanh(k\xi_l H_s)}{\tanh(k\xi_t H_s)} - 4\xi_l\xi_t + \frac{\rho_f^0}{\rho_s^0}\left(1-\frac{V^0}{c}\right)^2 \frac{\xi_l}{\xi_f}\frac{c^4}{c_t^4}\frac{\tanh(k\xi_f H_f)}{\tanh(k\xi_t H_s)} = 0. \tag{10}$$

2.4. Derivation of the Dispersion Relations

The dispersion relations are derived using impedance-type concepts (or admittance, which is the inverse of impedance). Consider the layer of solid of thickness $2H_s$, subject to a normal stress σ^{33} on either face, such that the motion is either symmetric or antisymmetric. The shear stress on both faces is zero. Similarly, consider the moving fluid layer of thickness $2H_f$ subject to a pressure disturbance p, again either symmetric or antisymmetric. The symmetry or antisymmetry implies that we need only consider half of the unit period of the system; that is, the solid and fluid half layers in $-H_s < x^3 < 0$ and $0 < x^3 < H_f$, respectively. Define the effective impedances,

$$Z_f^{(\alpha)} \equiv \frac{p}{v_f^3}\bigg|_{x^3=0}, \qquad Z_s^{(\alpha)} \equiv \frac{\sigma^{33}}{v_s^3}\bigg|_{x^3=0}, \tag{11}$$

where $\alpha = \pm 1$ indicates the symmetry. Thus, $\alpha = 1$ and -1 correspond to the symmetric and antisymmetric configurations, respectively.

Dispersion equations for guided waves can be deduced by imposing the force and velocity continuity conditions at the interface. The former are

$$\sigma^{33} = -p \qquad \text{and} \qquad \sigma^{31} = 0 \qquad \text{at } x^3 = 0. \tag{12}$$

The kinematic continuity condition on the normal velocities is determined from equation (3f) using the assumed dependence $e^{i(kx^1 - \omega t)}$, to give

$$v_f^3 = \left(1 - \frac{V^0}{c}\right) V_s^3 \qquad \text{at } x^3 = 0. \tag{13}$$

Thus, the dispersion relation in the presence of flow follows from equations (11) through (13) as

$$Z_s^{(\alpha)} + \left(1 - \frac{V^0}{c}\right) Z_f^{(\alpha)} = 0, \qquad \alpha = \pm 1. \tag{14}$$

The fluid and solid impedances can be found by considering motion in isolated slabs of either material. For convenience, the slabs may be repositioned with their centerlines along $x^3 = 0$, so that the solutions in each display parity with respect to x^3. The fluid impedance is determined by evaluating the ratio in the first of equations (11), at the bottom surface of the fluid slab, and a simple calculation based on equations (7) and (3c) gives

$$Z_f^{(\pm 1)} = -i \frac{\rho_f^0 c}{\xi_f} \left(1 - \frac{V^0}{c}\right) [\coth(k\xi_f H_f)]^{\pm 1}. \tag{15}$$

The impedance of the solid is obtained by taking the ratio in the second of equations (11) at the top surface of the solid slab, subject to the zero shear condition of the second equation (12). For simplicity, we assume the solid layer to be isotropic, in which case standard analysis gives

$$Z_s^{(\pm 1)} = i \frac{\rho_s^0 c_t^4}{c^3 \xi_l} \left(4\xi_l \xi_t (\coth(k\xi_t H_s))^{\pm 1} - \left(2 - \frac{c^2}{c_t^2}\right)^2 [\coth(k\xi_l H_s)]^{\pm 1} \right). \tag{16}$$

The dispersion relations of equations (8) and (10) now follow directly from equations (14) through (16).

3. ASYMPTOTIC LIMITS

We now consider several asymptotic cases of the general results defined by equations (8) and (10). The results and their interpretation are simplified by the introduction of dimensionless quantities. As noted in Table 1, all speeds are rendered dimensionless with respect to the shear speed c_t. Thus, $s_l = c_l/c_t$, $s_f = c_f/c_t$, etc., and the nondimensional speed of the guided wave is $s = c/c_t$. We also define a Mach number, M, a density ratio, δ, and a thickness ratio h,

$$M = \frac{V^0}{c_t}, \qquad \delta = \frac{\rho_f^0}{\rho_s^0}, \qquad h = \frac{H_f}{H_s}. \tag{17}$$

3.1. Short Wavelength Asymptotics: Two Half Spaces

In the short wavelength limit $|kH_s| \sim |kH_f| \gg 1$, the dispersion equations for both modes lead to the same equation:

$$(2 - s^2)^2 - 4\sqrt{1 - s^2}\sqrt{1 - s^2/s_l^2} + \delta s^2 (s - M)^2 \frac{\sqrt{1 - s^2/s_l^2}}{\sqrt{1 - (s - M)^2/s_f^2}} = 0. \tag{18}$$

This reduces to the Rayleigh equation for waves on a traction-free half space when

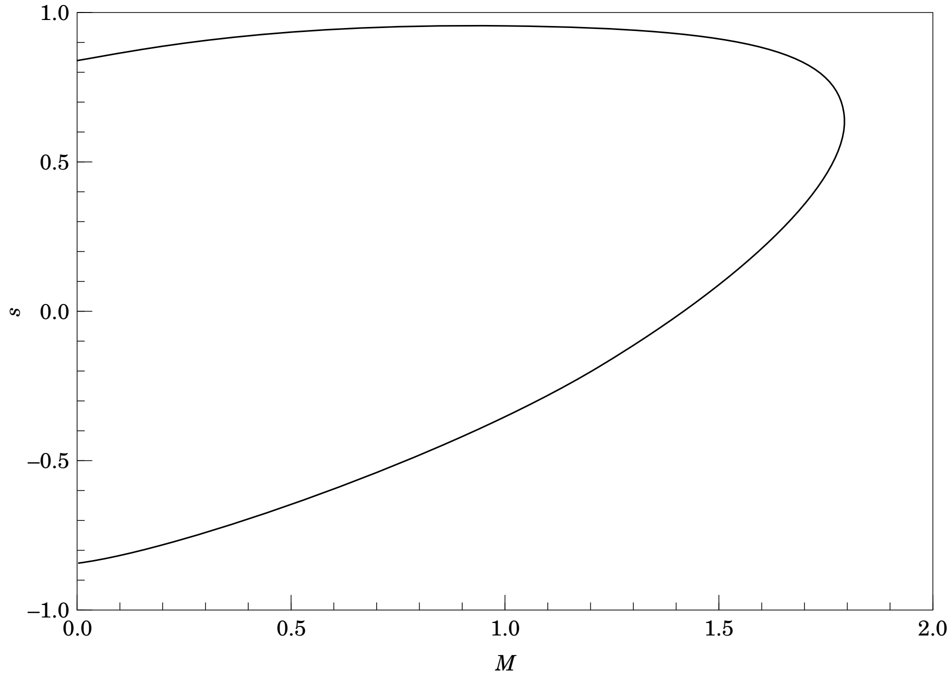

Figure 1. The real roots for flow over a solid half space, from equation (18). For simplicity, we have taken the fluid and solid as incompressible ($s_l, s_f \to \infty$), and $\delta = 1$. The emergence of complex roots occurs at $M \approx 1{\cdot}793$ for this case.

$\delta = 0$ (Rayleigh 1885; Achenbach 1973), and it becomes the Scholte equation for interfacial waves between fluid and solid half-spaces when $M = 0$ (Scholte 1948; 1949). By solving it numerically one can find that the roots for the interface wave speed become complex and the system becomes unstable for $M = \mathcal{O}(1)$. For example, Figure 1 shows the merging of two real roots as M is increased from zero. In this example, complex roots appear for M greater than about 1·793.

3.2. Long Wavelength Asymptotics: Symmetric Mode

The long wavelength asymptotics $|kH_s| \sim |kH_f| \ll 1$ are quite distinct for the symmetric and antisymmetric modes. For the symmetric mode we get

$$(2 - s^2)^2 - 4\left(1 - \frac{s^2}{s_l^2}\right) + \frac{\delta}{h} s^2 (s - M)^2 \left[\frac{1 - s^2/s_l^2}{1 - (s - M)^2/s_f^2}\right] = 0. \tag{19}$$

This can be rewritten as a fourth-order polynomial equation for s,

$$[(s - M)^2 - s_f^2](s^2 - s_p^2)\frac{\phi}{\rho_f^0 s_f^2} + (s - M)^2 (s^2 - s_l^2)\frac{(1 - \phi)}{\rho_s^0 s_l^2} = 0, \tag{20}$$

where $\phi = H_f/(H_f + H_s)$ is the porosity, and $s_p = c_p/c_t$, in which c_p is the speed of a longitudinal "plate" wave (see Table 1). Thus, $s_p^2 = 2/(1 - \nu)$. We note that the symmetric waves for the long wavelength asymptotic limit of an isolated plate in a fluid, $|kH_s| \ll 1$, $|kH_f| \to \infty$, are contained in equation (20) as the limiting case $\phi \to 1$.

At $M = 0$, equation (20) becomes a quadratic equation with respect to s^2 which has two physically meaningful roots: the velocities of the so-called Biot "fast" and "slow" waves (Rytov 1956; Schoenberg 1983, 1984; Plona *et al.* 1987). The speeds of the fast and slow waves are independent of the propagation direction for $M = 0$. However, any non-zero relative velocity of the fluid destroys the equivalence of the opposite directions. For small M, the last equation allows one to find the magnitude of the velocity splitting for each type of wave. Thus, letting s_0 (positive or negative) be the nondimensional wave speed for $M = 0$, fast or slow, and letting s_1 be the other speed, slow or fast, then the split velocities are

$$s = s_0 + \left[\frac{(s_0^2 - s_p^2)(\phi/\rho_f^0 s_f^2) + (s_0^2 - s_l^2)[(1-\phi)/\rho_f^0 s_l^2]}{(s_0^2 - s_1^2)[\phi/\rho_f^0 s_f^2 + (1-\phi)/\rho_f^0 s_l^2]}\right] M + \mathcal{O}(M^2). \tag{21}$$

The fast and slow wave roots simplify when both constituents are incompressible, that is, $s_l,\ s_f \to \infty,\ s_p = 2$. The fast speed then becomes infinite, but the slow roots are

$$s = \pm \left[\left(\frac{h}{\delta + h}\right)\left(4 - \frac{M^2\delta}{\delta + h}\right)\right]^{1/2} + \frac{M\delta}{\delta + h}. \tag{22}$$

The critical Mach number is explicit in this case. That is, the slow wave speeds become complex at $M = 2\sqrt{1 + h/\delta}$, one being associated with an unstable disturbance.

3.3. Antisymmetric Modes: Flexural Waves

In the asymptotic limit of $|kH_s| \ll 1$, the dispersion equation (10) for the antisymmetric mode gives, to leading order,

$$s^2 - s_b^2 + \delta h (s - M)^2 \frac{\tanh(kH_f \xi_f)}{kH_f \xi_f} = 0. \tag{23}$$

Here s_b is the nondimensional phase speed of a flexural wave (or bending wave) on a plate *in vacuo*: $c_b = c_t\, |kH_s| \sqrt{2/3(1-\nu)}$. Thus, $s = \pm s_b$ is recovered from equation (23) with $\delta = 0$. The bending wave is dispersive, and, by assumption, much slower than the shear wave. However, we have retained the parameter ξ_f in (23) rather than set it to unity, in order to be consistent with standard analyses for fluid-loaded plates, e.g. Junger & Feit (1986). The dispersion relation for a fluid-loaded plate in the absence of flow is obtained from equation (23) by setting $M = 0$.

If the fluid layer is also very thin compared with the wavelength, i.e. $|kH_f| \ll 1$, then equation (23) reduces to a quadratic equation in s, yielding

$$s = \pm \frac{s_b}{1 + \delta h} \sqrt{1 + \delta h (1 - \mathrm{m}^2)} + \frac{M \delta h}{1 + \delta h}, \tag{24}$$

where m is the Mach number relative to the bending wave phase speed:

$$\mathrm{m} = \frac{V^0}{c_b} = \frac{M\sqrt{3}}{|kH_s|\, s_p}. \tag{25}$$

When $M = 0$, the roots yield $s = \pm s_b/\sqrt{1 + \delta h}$, which correspond to flexural waves on an isolated plate which has the bending stiffness of a single elastic layer, and the mass of a single period of the solid–liquid system. That is, the effect of the fluid is just an added mass as it moves in phase with the flexural motion. For small values of M, or equivalently m, the two roots are regular perturbations of the flexural wave roots. For large m, on the other hand, we have the possibility of two complex roots when the discriminant of equation (24) goes to zero. The existence of complex-conjugate roots indicates that the flow causes a flutter-like instability. This occurs for $\mathrm{m} > \mathrm{m}_c$, where the critical value is

$$\mathrm{m}_c = \sqrt{1 + (\delta h)^{-1}}. \tag{26}$$

In summary, antisymmetric disturbances of the layered system are unstable for very long wavelength ($\mathrm{m} \gg 1$ or $|kH_s| \ll M$), but short wavelengths ($\mathrm{m} \ll 1$) are stable. There is a critical wavelength $k = k_c$ above which all disturbances are unstable, and it is defined by $\mathrm{m} = \mathrm{m}_c$, as

$$k_c = \frac{V^0}{c_t H_s} \sqrt{\frac{3(1-\nu)}{2(1+\rho_s^0 H_s/\rho_f^0 H_f)}}. \tag{27}$$

This is premised on the assumption that both $k_c H_s$ and $k_c H_f$ are small.

Finally, in order to analyse flutter of an isolated elastic layer, we let $|kH_f| \to \infty$ in the dispersion equation (23) for the antisymmetric mode. Using the same variables as before, we consequently obtain the following equation:

$$s^2 - s_b^2 + \frac{\delta(s-M)^2}{|kH_s|\ \xi_f} = 0. \tag{28}$$

When the fluid is incompressible, this equation becomes a quadratic with roots

$$s = \pm \frac{s_b\,|kH_s|}{\delta + |kH_s|} \sqrt{1 + \frac{\delta(1-\mathrm{m}^2)}{|kH_s|}} + \frac{M\delta}{\delta + |kH_s|}, \qquad \text{incompressible fluid.} \tag{29}$$

The possibility of complex conjugate roots again shows that flutter instability occurs for long wavelength perturbation of the system. That is, the system is stable (unstable) for $|k| > k_c$ ($|k| < k_c$), where the finite wavenumber defining the onset of the flutter regime is

$$k_c = \frac{\delta}{H_s}\lambda, \tag{30}$$

and λ is the unique positive root of

$$\lambda^3 + \lambda^2 = (1-\nu)\frac{3}{2}\frac{M^2}{\delta^2}. \tag{31}$$

Brazier-Smith & Scott (1984) studied the stability of wave solutions for a thin plate in an incompressible flow. Subsequently, Crighton & Oswell (1991) discussed the response of the same system to a line drive on the plate, and further analysed the stability issue. These studies are concerned with the temporal and frequency behavior, and they therefore require expressions for the wavenumber k in terms of the frequency ω. The roots defined by equation (29) provide ω as an explicit function of k, and are much simpler to deal with as compared with the inverse functional relations defined by the five roots for k in terms of ω.

4. SUMMARY

The flow of compressible fluid through a layered medium provides a very rich system for studying the phenomenon of acoustics in fluid conveying structures. Starting from the exact nonlinear equation of motion we have derived the equations for small dynamic disturbances superimposed upon a steady flow configuration. The possible wave types for the periodically layered medium may be distinguished by their parity, symmetric and antisymmetric, each of which displays quite distinct stability characteristics.

The general dispersion relation for symmetric modes is given by equation (8), with short and long wavelength limits in equations (18) and (20), respectively. It is found that instability is possible only for flow speed on the order of the bulk wave speeds, i.e. for $M = \mathcal{O}(1)$.

Regarding antisymmetric modes, the general dispersion relation of equation (10) has the same short wavelength limit as that for symmetric modes, equation (18). Thus, disturbances of short wavelength become unstable only for $M = \mathcal{O}(1)$. However, disturbances of very long wavelength are potentially unstable in the presence of flow for both the periodic system, as indicated by equation (24), and for an isolated plate, from equation (29). The associated long wavelength wave types are analogous to flexural waves on plates *in vacuo*. For a given flow speed, or M, there is a critical wavenumber, k_c, such that quasi-flexural waves are stable for $k > k_c$, but instability is possible for all $k < k_c$.

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